J. Fluid Mech. (1968), *vol.* 34, *part* 3, *pp.* 449–464
Printed in Great Britain

The dynamic response of towed thermometers

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(Received 9 September 1967 and in revised form 6 July 1968)

Small-scale temperature fields in water were used to test the dynamic response of towed thermometers of the platinum film resistance type. Laminar buoyant plumes rising from submerged heaters below the line of motion were the test temperature fields. The analysis of results was based on an approximate 'diffusion-layer' model of the dynamic heat-transfer process occurring near the platinum film on the probe tip. The model represents the linear heat transfer into a two-layer semi-infinite medium, with the platinum thin film located at the interface between a water layer of thickness Δ and a semi-infinite substrate of glass. The differences of the thermal properties of water and glass were found to be negligible. The characteristic time Δ^2/D, where D is the thermal diffusivity of water, was determined by the ratio of actual to film-indicated plume-peak temperature, assuming a sinusoid approximation to the plume profile. The frequency response for the same operating conditions as the plume tests could then be obtained from the diffusion-layer model.

1. Introduction

Theoretical or experimental determination of the frequency response of thermometers, held fixed in a fluid stream or towed through a fluid body of non-uniform temperature, has seldom been attempted. Because of the complexity of the forced-convection heat-transfer process, an experimental test of frequency response, or a test of dynamic response that can be interpreted in terms of frequency response, is desirable. The response of platinum film thermometers of the type used by Grant, Hughes, Vogel & Moilliet (1968) is of main interest, but the test technique described can be applied to other types as well.

Two reasons for imperfect frequency response of the platinum film type of thermometer are apparent. If the probe material is not a perfect insulator, the amplitude of film-temperature fluctuations will be attenuated by the thermal inertia of this substrate. In addition, amplitude attenuation occurs in a thin thermal boundary layer next to the probe surface because of the deformation of the temperature-field structure caused by the probe. Figure 1 shows how material surfaces in an incompressible fluid are deformed as the surfaces are swept over the probe tip (cf. Lighthill 1956). Along the streamline approaching the stagnation point, there is longitudinal compression (along the flow direction) and radial elongation. A longitudinal temperature-fluctuation field in the free stream must undergo a similar deformation leading to increasing temperature gradients. As a result, there will be increasing attenuation by heat conduction of temperature-fluctuation amplitude as the probe tip is approached.

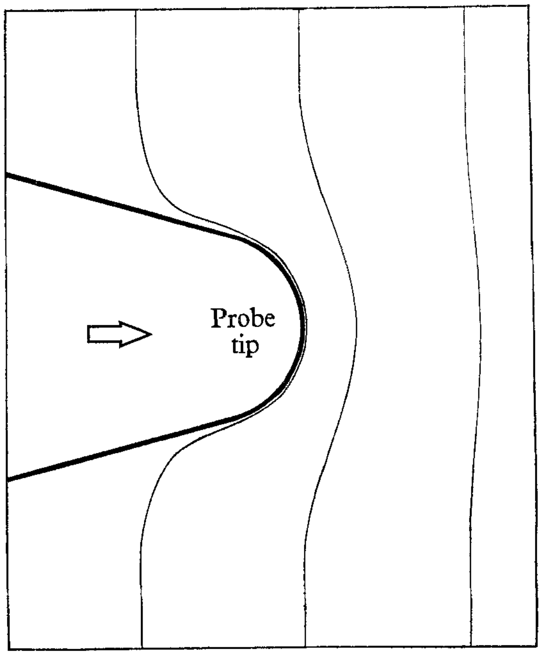

FIGURE 1. Deformation of fluid material surfaces by a moving probe.

The need for frequency-response corrections to oceanic thermal microstructure measurements is also apparent. Experimental results, which will be described later, show that for a platinum film probe the frequency response F is approximately

$$F = \exp[-\Delta(\pi f/D)^{\frac{1}{2}}], \tag{1.1}$$

where f is the frequency, D the thermal diffusivity of the fluid, and Δ a length of the order of the velocity boundary-layer displacement thickness over the film, δ_1. If a minimum value of δ_1 is taken as that at the stagnation point on a spherically tipped probe with tip radius L, it can be shown that the frequency response correction to a measured temperature-fluctuation power spectrum will be about 3 dB at the wave-number $1/\sigma L$, where σ is the Prandtl number (ν/D) and ν is the kinematic viscosity. $1/\sigma L$ can be compared with an upper wave-number of interest, viz. the characteristic wave-number of Batchelor (1959), $(\epsilon/\nu^3)^{\frac{1}{4}}\sigma^{\frac{1}{2}}$, where ϵ is the turbulence energy dissipation rate. Typical oceanic values of σ and ν are 9 and 0·013 cm²/sec. Munk & Macdonald (1960) estimate a mean value of ϵ in the oceans, due to tidal energy dissipation alone, of 10^{-5} cm²/sec³. For these values, $1/\sigma L$ will be less than $(\epsilon/\nu^3)^{\frac{1}{4}}\sigma^{\frac{1}{2}}$ for $L > 0{\cdot}025$ cm.

Thus frequency-response corrections are expected to be necessary for oceanic thermal microstructure measurements with present probes. A plume-test technique for determining the dynamic response of towed platinum film probes is described in § 2. The analysis of plume-test results with a diffusion-layer model, the corresponding prediction of frequency response for the plume tank operating conditions, and some results for diffusion-layer thickness are given in § 3. Finally, in the appendix by B. A. Hughes, the scaling of plume-test results to other operating conditions is considered and the accuracy of the diffusion-layer model is proven by the direct computation of frequency response from plume-test power spectra.

2. Experimental method

To test the dynamic response of the towed temperature probe, it was passed through a known temperature field of suitably small scale by towing it through a laminar buoyant plume rising from a submerged heater (cf. Forstrom & Sparrow 1966).

The plume tests were conducted in a miniature towing tank approximately 160 cm long and 15 by 15 cm in cross-section. The test probe was carried through the plume by a thin strut attached to a towing platform that slid along taut horizontal wires above the water surface. Towing power was provided by a falling weight. Probe speed was measured by means of an electrical signal from a fixed contact that touched an interrupter strip on the moving carriage. A swinging cable connected the probe to its bridge, the output voltage of which indicated film temperature.

A quiet plume was essential to good repeatability of the results. The state of the plume was monitored by a simple schlieren system, with the light beamed through the Plexiglas tank walls. Figure 2, plate 1, shows a typical plume from a heater wire stretched across the tank. The camera was aimed along the wire, which is behind the end of the electrical lead on the tank wall. Because it was necessary that the wire be nearer one end of the tank than the other, the plume curves toward the tank centre. Tiny dust and lint particles that are caught up in such plumes showed that the maximum convection speeds were only a few cm/sec. Since the towing speeds varied from 50 to 150 cm/sec, it was considered that the boundary layer about the probe tip was not affected significantly by the plume.

The plume temperature profile was measured with the thermistor bead shown in figure 2. The tiny bead was supported by the compound rest of a jeweller's lathe so that it could be moved slowly and steadily through the plume along the expected probe trajectory. Temperature-versus-horizontal-distance was obtained by recording the thermistor bridge unbalance with a pen recorder while turning the rest screw in synchronization with flashes of the recorder timer lamp. Bead speed was reduced until only negligible changes in the indicated profile were obtained.

Wire heaters were used in order to have approximately two-dimensional plumes, for which lateral deviation of the probe from the thermistor bead path would produce no error. Typical plume-temperature profiles at various heights above a wire heater are shown in figure 3. For good repeatability of plume profiles, it was necessary that the wire be clean and that the water be (i) nearly lint-free,

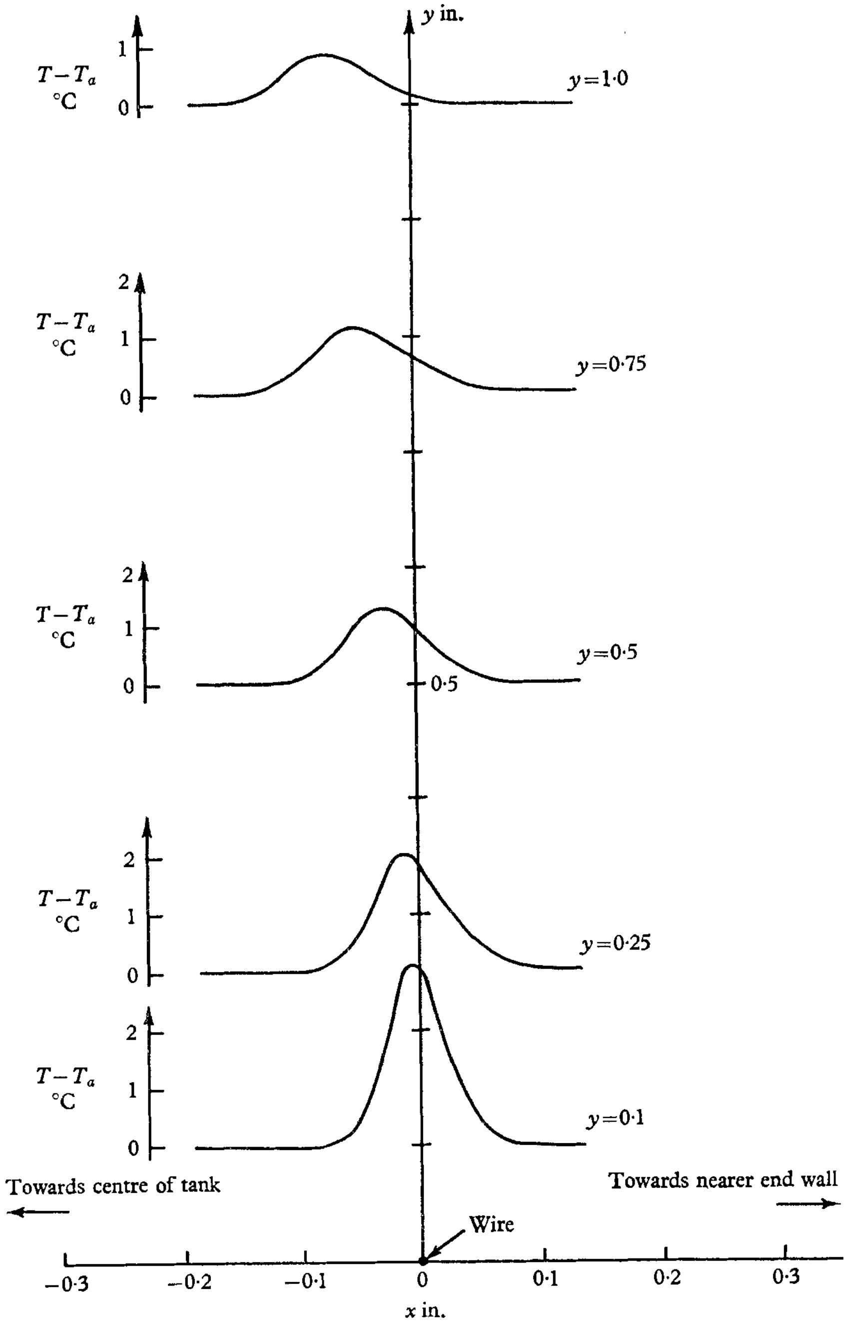

FIGURE 3. Plume temperature profile at various levels above the wire heater.

(ii) at room temperature, and (iii) free of excess air content (to reduce bubble formation).

Both the probe speed and the temperature fluctuation of the film as it passed through the plume were obtained from oscilloscope traces of the interrupter strip voltage and the probe bridge output voltage. Figure 4, plate 2, is a typical oscilloscope record. The known horizontal spacing of the conductive strips, corresponding to the steps in the interrupter trace, as well as the known sweep rate, determine the probe speed. Because the static temperature sensitivity was 263 mV/°C and the beam sensitivity was 20 mV/scale cm in this case, the bridge output trace shows that the amplitude of the film-indicated temperature rise was 0·40 °C. Since the plume-peak temperature rise for this test was 1·14 °C (according to the thermistor survey), the ratio of film-indicated to actual plume-peak amplitudes was 0·35. The notable trailing off of the bridge output trace is predicted by the diffusion-layer model, as will be seen later.

3. Analysis and results

In the plume tests, the probe passes through known temperature fields of controllable scale. The probe bridge output voltage is assumed to give the instantaneous temperature of the film for the highest frequencies of interest. The imperfect indication of the plume temperature profile is assumed to be due solely to the characteristics of the heat-transfer process from the free stream to the probe tip.

If the temperature fluctuations are small, the heat-transfer process can be shown to be equivalent to a linear dynamic system. The input to this system is the actual plume temperature profile, which is seen by the moving probe as a function of time. The output of the system is the film-indicated temperature profile. Thus the dynamic response of the film may be described by a Laplace transfer function, which could be determined from a suitable collection of input and output temperature profiles. This general approach, however, was not considered appropriate in the exploratory work described here; it was considered more desirable to interpret the experimental data in terms of a simplified, approximate model of the heat-transfer process.

The approximate model was based on the flow situation shown in figure 1, which suggests that the final stage of heat transfer can be approximated by linear Couette flow with heat conduction normal to the flow. Because the heat flux and the flow are perpendicular, the model literally represents unsteady linear heat conduction into a layered, effectively solid medium of water, film and substrate. Note, however, that the characteristic time of the platinum film of thickness l and thermal diffusivity D_p is l^2/D_p. Because of the extreme thinness of the platinum, this characteristic time is so small that the thermal inertia of the film can be ignored for the frequencies of interest in this work. Thus the heat flux and temperature are taken to be continuous across the film on the plane $z = 0$, with a layer of effectively stagnant water of thickness Δ above and a substrate of glass below. The film temperature is $T(0,t)$ and the input temperature is $T(\Delta,t)$. When the unknown thickness Δ of this diffusion-layer model has been chosen by means of the plume test, the model then provides a description of the dynamic response of the temperature probe.

The full statement of the diffusion-layer model is contained in the governing equations and boundary conditions of linear heat conduction. Because of the small temperature range of interest, the heat conductivities and thermal diffusivities of the fluid (k_f, D) and the substrate (k_s, D_s) are effectively constant. Thus the governing equations are

$$0 < z < \Delta, \quad \partial T/\partial t = D(\partial^2 T/\partial z^2), \tag{3.1}$$

$$z < 0, \quad \partial T/\partial t = D_s(\partial^2 T/\partial z^2), \tag{3.2}$$

and the boundary conditions are

$$T(\Delta, t)\ \text{(given)}, \tag{3.3}$$

$$T(0+) = T(0-), \tag{3.4}$$

$$k_f(\partial T/\partial z)_{0+} = k_s(\partial T/\partial z)_{0-}, \tag{3.5}$$

$$z \to -\infty, \quad T(z,t) \to 0. \tag{3.6}$$

The last condition amounts to declaring the substrate semi-infinite. This is a valid approximation as long as the characteristic length of the exponential decay of temperature fluctuations with depth in the substrate is much smaller than, say, the radius of the probe tip at the film. Since that characteristic length is $(D/\pi f)^{\frac{1}{2}}$, the semi-infinite substrate approximation breaks down as $f \to 0$. However, this error is unimportant because the response is nearly unity at low frequencies.

If there is a sinusoidal temperature fluctuation of frequency f and amplitude A_Δ on $z = \Delta$, then the relative amplitude and phase lag of the film temperature, A/A_Δ and ϕ, from (3.1) to (3.6) are

$$A/A_\Delta = [(\cosh d + K \sinh d)^2 + (K^2 - 1)\sin^2 d]^{-\frac{1}{2}}, \tag{3.7}$$

$$\phi = \tan^{-1}\left[\left(\frac{K + \tanh d}{1 + K \tanh d}\right)\tan d\right], \tag{3.8}$$

with $\quad d \equiv \Delta(\omega/2D)^{\frac{1}{2}}, \quad \omega \equiv 2\pi f \quad \text{and} \quad K \equiv (k_s/k_f)(D/D_s)^{\frac{1}{2}}.$

With a water layer over a glass substrate, the thermal properties parameter of the interface, K, is about 0·9. If $K = 1{\cdot}0$, (3.7) and (3.8) reduce to

$$A/A_\Delta = \exp(-\phi), \tag{3.9}$$

$$\phi = \Delta(\omega/2D)^{\frac{1}{2}}. \tag{3.10}$$

These are also the results for the homogeneous semi-infinite solid with the thermal diffusivity D. The effects on the plume-test frequency-response prediction that arise if K is taken to equal 1·0 when it should be 0·9 were estimated by considering the cases of $K = 1{\cdot}0$ and $K = 0$. It was found that the frequency-response function would be in error by about 6 % over the A/A_Δ range of interest; however, this error was considered negligible.

Since $K = 1{\cdot}0$ makes the film frequency response and phase lag the same as for the case of a homogeneous medium with the diffusivity of the upper layer, the analogous relation must also apply to the film's response to an arbitrary input $T(\Delta, t)$. Thus, if $T(z, 0) = 0$ for $z \leqslant 0$, the film temperature for $t > 0$ is

$$T(0,t) = (2/\sqrt{\pi})\int_{u_0}^{\infty} T(\Delta, t-\tau)\exp(-u^2)\,du, \tag{3.11}$$

for $\quad \tau \equiv \Delta^2/4D \quad \text{and} \quad u_0 \equiv \Delta/2(Dt)^{\frac{1}{2}}$ (Carslaw & Jaeger 1959).

Since the probe speed W was always essentially constant while the probe passed through the plume, an experimental profile of excess temperature $T(x) - T_a$ (figure 3) was converted to a temperature fluctuation input $T(\Delta, t)$ using $dx = W\,dt$. It was found that the typical input profile could be described satisfactorily by a sinusoidal single wave, with

$$T(\Delta, t) = 0 \quad \text{for} \quad \omega t < 0; \; 2\pi < \omega t,$$

$$T(\Delta, t) = A_\Delta(1 - \cos \omega t)/2 \quad \text{for} \quad 0 < \omega t < 2\pi.$$

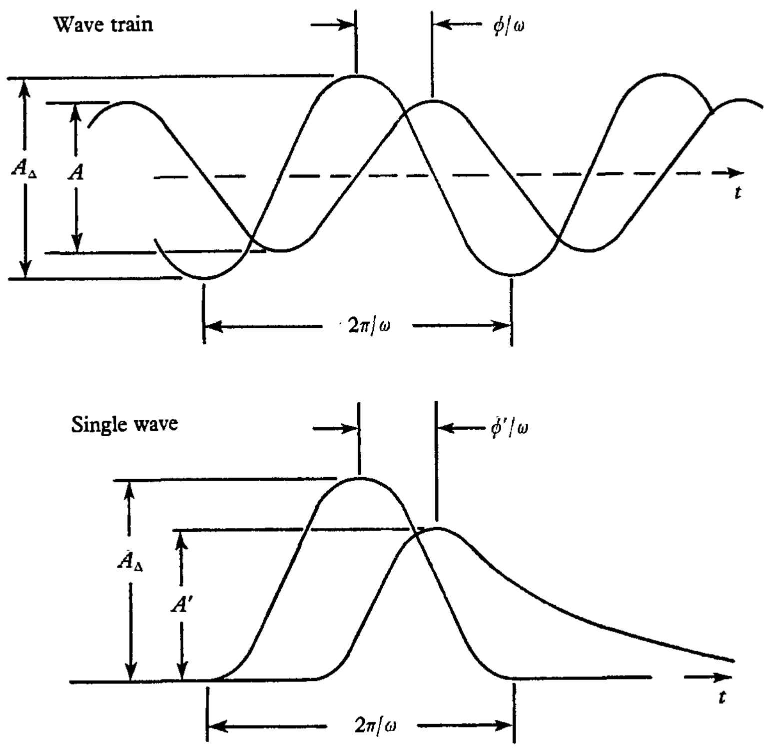

FIGURE 5. Definitions of amplitudes A_Δ, A and A', and phase lags ϕ and ϕ'.

For this input $T(0, t)$ was calculated numerically by computer with (3.11) for various values of $\omega\Delta^2/D$ and a sequence of ωt values. From the resultant plots of $T(0, t)$, the amplitude ratio of the output peak A'/A_Δ and its lag time and lag angle ϕ' relative to the input peak were determined. Definitions of A' and ϕ' are shown in figure 5. Figures 6 and 7 give the plots of A/A_Δ, A'/A_Δ, ϕ and ϕ' for the diffusion-layer model with $K = 1{\cdot}0$.

The single-wave profile was fitted to the plume test profile by matching the peak amplitude and a 'wavelength', defined as twice the mid-height width of the plume profile. Thus each plume test was assigned a 'frequency', $f = W/\text{wavelength}$.

The actual plume temperature profiles are broader at the base than the fitted

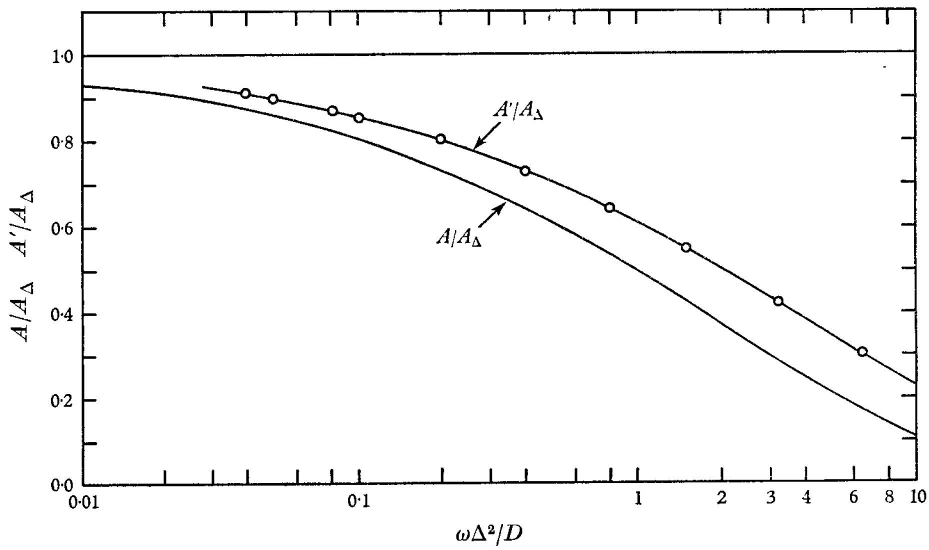

FIGURE 6. Amplitude ratios at depth Δ for heat conduction into a semi-infinite homogeneous medium.

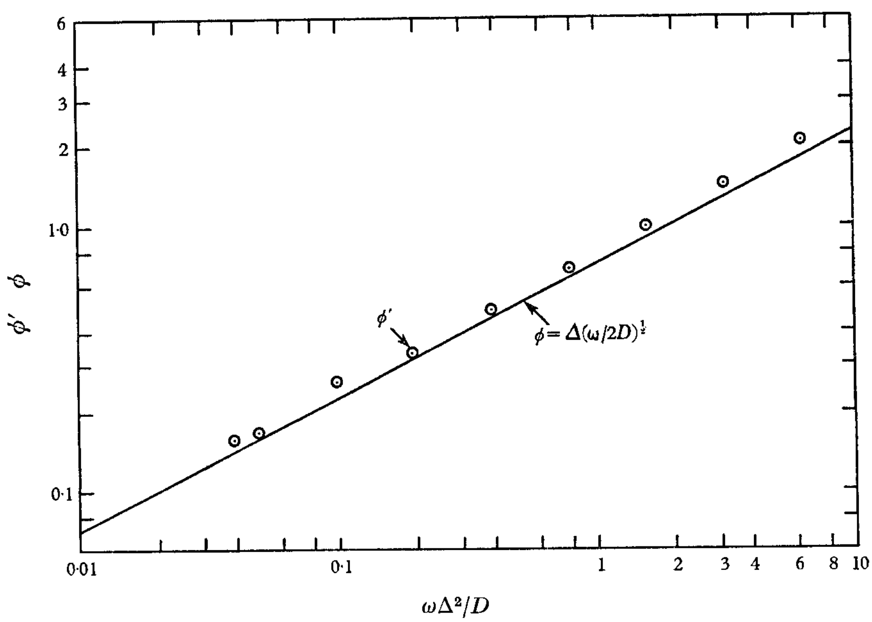

FIGURE 7. Phase lags at depth Δ for heat conduction into a semi-infinite homogeneous medium.

sinusoidal single-wave profiles. To check on the sensitivity of the results to this difference, A'/A_Δ and ϕ' were also calculated, using the diffusion-layer model with $K = 1{\cdot}0$ for a 'power law' input profile that was considered to allow a better fit to typical plume profiles. In this instance

$$T(\Delta, t) = 0 \quad \text{for} \quad t < 0;\ 2t_1 < t,$$

$$T(\Delta, t) = A_\Delta\left[1 - \left|\frac{t-t_1}{t_1}\right|^{\frac{3}{2}}\right]^2 \quad \text{for} \quad 0 < t < 2t_1.$$

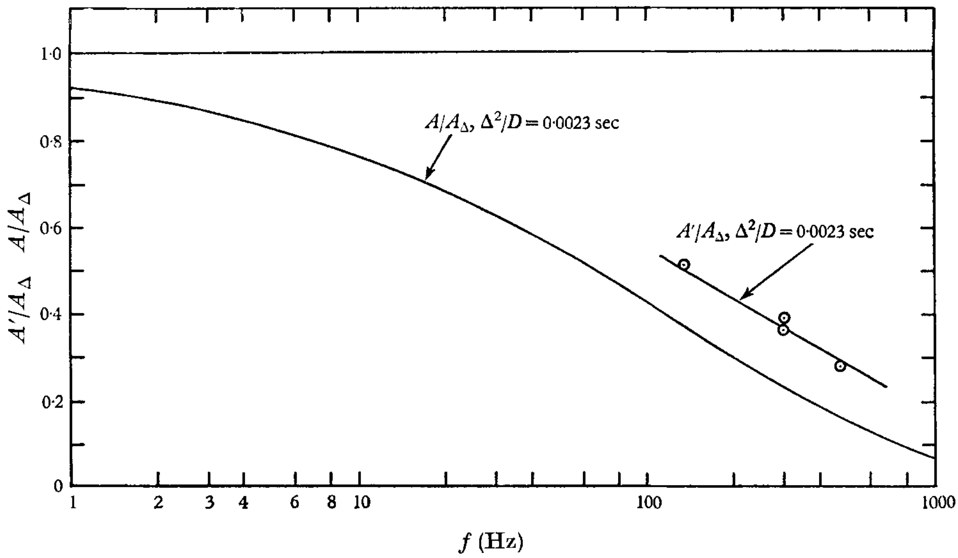

FIGURE 8. Determination of the characteristic time Δ^2/D by fitting the A'/A_Δ relation for the diffusion-layer model with $K = 1{\cdot}0$ to plume test data. The corresponding predicted frequency response $A/A_\Delta(f)$ is also shown.

The fitting technique described above results in $1/f = 1{\cdot}76t_1$. For typical experimental A'/A_Δ values of approximately 0·3 to 0·5, it was found that the resultant estimate of the characteristic time Δ^2/D was about 5 % lower, owing to the use of the more accurate profile. Because the experimental uncertainty was roughly of the same order, this difference was ignored.

When a set of experimental values of A'/A_Δ for a range of plume widths, along with corresponding f values, had been obtained for a particular probe, towing speed, and tank fluid, a semilog plot similar to figure 8 was prepared to the same scale as figure 6. By overlaying the two figures, the theoretical curve of A'/A_Δ versus $\omega\Delta^2/D$ was fitted to the experimental values of A'/A_Δ versus f. When a good fit was obtained with the A'/A_Δ scales aligned, the experimental behaviour was consistent with the theoretical model and the value of Δ^2/D was determined by the ratio of the f and $\omega\Delta^2/D$ scales. The corresponding frequency-response curve for the operating conditions employed in the plume tests is then predicted by (3.9), as shown by the A/A_Δ curve in figure 8.

It was concluded from the power law profile input that the signals from the towed probe should not be very sensitive to small departures from the assumed sinusoid plume profile. This being so, it would not be necessary to solve (3.11) for each individual profile. This assumption, however, was so important to the argument that it was worth while to carry out the complete calculation at least once for a real plume profile. This exercise also served as a partial test of the validity of the diffusion-layer model.

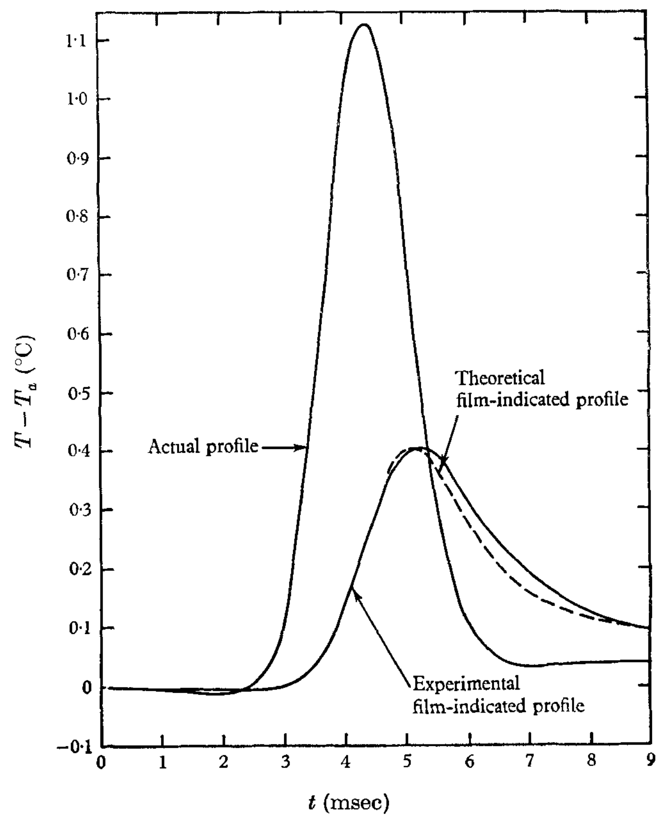

FIGURE 9. Comparison of actual, experimental film-indicated, and predicted film-indicated plume temperature profiles for a plume test.

Using the thermistor-indicated profile of $T(x) - T_a$ for the test which gave the oscilloscope trace in figure 4, the theoretical film-indicated profile was calculated by means of (3.11), using $dx = W\,dt$ and $\Delta^2/D = 0{\cdot}0023$ sec (from figure 8). The observed and theoretical results are shown in figure 9. Because it was difficult to determine the exact instant at which the probe reaches the plume, the rise of the experimental film-indicated profile was arbitrarily matched to the rise of the theoretical film-indicated profile. The good agreement of the theoretical and experimental peak heights verifies the sinusoid approximation of the true plume profile. The overall agreement of the profile shapes would seem to be as good as could be expected, in view of the approximations made in the diffusion-layer model. In later work, reported in the appendix, it became possible to eliminate the assumption of the diffusion-layer model by obtaining the frequency response for a broad frequency range directly from power spectra of plume test profiles. That work shows that the diffusion-layer model appears to be correct for within 5 % up to about 625 Hz for a typical conical probe.

Using the plume-test technique, experimental values of Δ^2/D for several platinum film temperature probes of conical, wedge, and parabolic cylinder shapes were determined and compared for a standard speed and water temperature. There was no evidence of systematic deviation from diffusion-layer behaviour for the available range of plume widths. The values of Δ were found to roughly equal the calculated values of δ_1, the laminar boundary-layer displacement thickness over the film centre. This behaviour and the suitability of the diffusion-layer model were later corroborated (Fabula 1968) by the theoretical frequency response of stagnation-region films such as those on the wedge and parabolic cylinder probes.

All of the above platinum film probes had uncoated films. Recently probes with their platinum film coated with a thin layer of quartz, for electrical insulation from sea water, have been introduced. Some tests with such a probe are discussed in the appendix. It can be shown that a uniform quartz coating over a platinum film on a glass substrate affects the dynamic response of the uncoated film merely by increasing the diffusion layer thickness by about the coating thickness. Thus the coating calls for no special treatment of plume test data.

A difference between the plume test results and the theoretical stagnation-region film behaviour was found with regard to the speed dependence of frequency response. Plume tests with an uncoated conical probe for a speed range of 75 to 150 cm/sec showed that Δ varied approximately as $W^{-0.32}$, instead of $W^{-0.5}$ as expected either from $\Delta \approx \delta_1 \sim W^{-0.5}$ or from the theory for the stagnation-region film. The experimental behaviour is attributed to the conical-probe film being outside the stagnation region. Speed dependence tests of a probe with a stagnation-region film have not been made as yet.

This study was made at the Defence Research Establishment Pacific, Victoria, B.C., under an interlaboratory exchange programme. Dr H. L. Grant suggested the problem and contributed much to the success of the work.

Appendix By B. A. Hughes, Defence Research Establishment Pacific, Victoria, B.C.

Prandtl number scaling

In principle, the frequency response of a given probe can be investigated with the plume test technique for the same towing speed, water temperature and salinity as in measurements of oceanic temperature fluctuations. In practice, however, this is awkward to do. It is preferable to test at standard conditions and to scale the frequency response to consider the variation of dimensionless parameters of the heat-transfer problem. It can be shown that the frequency response for a given probe and a fixed temperature-fluctuation wave-number should depend mainly on the probe-tip Reynolds number (R), the Prandtl number ($\sigma \equiv \nu/D$), and the interface thermal-properties parameter $K \equiv (k_s/k_f)(D/D_s)^{\frac{1}{2}}$. Fortunately, K and D are nearly constant over the water temperature and salinity ranges of interest. Since towing speed in the plume tank can be adjusted, the problem of scaling reduces to considering the variation of Reynolds and Prandtl numbers due to different values of ν. In the early work, only the dependence upon Reynolds number for fixed Prandtl number was investigated by the tests of the dependence of Δ upon towing speed. It has subsequently been necessary to determine the Prandtl number dependence experimentally in order to calculate the frequency response correction for the ocean turbulence measurements reported by Grant *et al.* (1968). These measurements were made in water at approximately 10 °C while it is convenient to perform the plume tests at 20 °C in order to minimize convection currents in the tank.

Using the method explained in the main body of this paper, the diffusion-layer thickness Δ was measured for a particular conical probe at essentially two different Prandtl numbers and the same probe Reynolds number.

The platinum film on this probe covers the hemispherical tip completely and extends part way back along the cone. Electrical connexions are made on opposite sides of the conical part (see figure 10, plate 3).

Different Prandtl numbers were obtained by changing the ambient temperature of the water in the plume tank. Fifty measurements in all were taken, twenty-one at σ near 6·6, corresponding to an ambient temperature near 20 °C, and twenty-nine at σ near 9, corresponding to an ambient temperature near 10 °C.

The velocity at which the platinum probe travelled through the plume was measured for each run and the individual Reynolds numbers were calculated (except for the exclusion of a constant length scale). The mechanism which controlled the probe velocity could be adjusted to give either a slow speed (near 134 cm/sec) which was used for the 20 °C runs or a fast speed (near 180 cm/sec) for the 10 °C runs. In this way major changes in the probe Reynolds number were eliminated. The minor variations in Reynolds number which still existed were allowed for by scaling the measured diffusion layer parameter as $\Delta R^{0\cdot 32}$ = constant, in accord with the speed dependence test result mentioned in the main body of the paper.

The fifty runs were actually performed over a period of 3 months, and during this time the experimental techniques were continually modified and improved, with the result that the scatter between the results of supposedly identical measurements decreased considerably. The improvement is believed to be chiefly due to greater care in the reduction of currents which contribute to instability of the plume. From the point of view of technique, the last two sets of runs are considered our 'best'. These are twelve runs, six at each ambient temperature.

Taking guidance from the theory of the frequency response of stagnation-region films (Fabula 1968) we have fitted a power law dependence of Δ on σ to these results. Using least squares and the form $\Delta\sigma^n$ = constant for constant Reynolds number, the information in table 1 was determined. The confidence limits on n were obtained using Student's t parameter.

Data	n	95% confidence limits
All 50 runs	0·30	$\pm$ 0·10
Final 12	0·30	$\pm$ 0·04

TABLE 1

Figure 11 shows $\log_e \Delta$ plotted against $\log_e \sigma$. The final 12 runs are represented by the encircled points and the best-fit power law to these points is shown. According to the stagnation-region film theory, the exponent n would have been about 0·33.

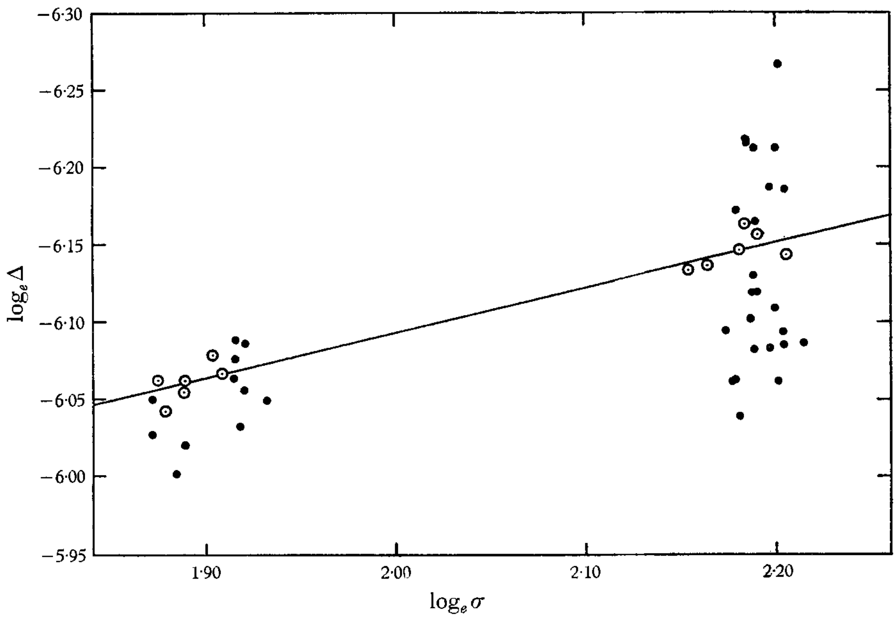

FIGURE 11. $\text{Log}_e \Delta$ *vs.* $\log_e \sigma$. The straight line has the equation $\log_e \Delta = -0{\cdot}30 \log_e \sigma - 5{\cdot}493$ and is a least-squares best fit to the encircled points.

Experimental measurement of platinum thermometer frequency response

The diffusion-layer model predicts the response of the probe at all frequencies, but the prediction is uncertain at frequencies far above or below the range corresponding to the range of usable plume wavelengths divided by speed. When the 50 measurements were made later for the determination of Prandtl number dependence, digital computer techniques were available for the computation of power spectra from which F can be obtained at all frequencies without assuming the diffusion-layer model. The signals obtained from the slow-speed thermistor and the film were recorded digitally on magnetic tape and power spectra were obtained for both instruments for each run. Let the signals, measured in °C, from the platinum thermometer and the thermistor be represented respectively by $S_p(t)$ and $S_{th}(t)$. Then

$$S_p(t) = \int_{-\infty}^{+\infty} F(f)\, A_p(f)\, G(f) \exp\{2\pi ift + i\phi(f)\}\, df,$$

$$S_{th}(t) = \int_{-\infty}^{+\infty} A_{th}(f)\, G(f) \exp(2\pi ift)\, df, \qquad \text{(A 1)}$$

where $\phi(f)$ is the phase response associated with F, and A_p and A_{th} are the frequency dependent gains of the amplifiers in the platinum thermometer and the thermistor circuits respectively. These gains are associated with amplifiers designed to pre-emphasize the high frequencies before the signals are spectrally analyzed. This reduces the errors at these frequencies due to digitizer and computer 'bit' noise.

The autocorrelations of the signals were formed for all possible lags and the power spectra were obtained by calculating the Fourier transforms of the autocorrelations (without smoothing). Let

$$P_p(f) = \left| \int_{-\infty}^{+\infty} S_p(t) \exp(-2\pi ift)\, dt \right|^2 \qquad \text{(A 2)}$$

with a similar expression for the thermistor. Then

$$F(f) = \left\{ \frac{P_p(f)}{P_{th}(f_1)} \right\}^{\frac{1}{2}} \frac{|A_{th}(f_1)|}{|A_p(f)|}, \qquad \text{(A 3)}$$

where f_1 is the frequency at which the spectral estimates were obtained from the thermistor signal multiplied by the ratio of the velocity of the platinum thermometer to the velocity of the thermistor. This refers f_1 and f to the same length scale in the plume.

Figure 12 shows four frequency response curves as obtained from (A 3). Each of these curves was chosen from our final twelve 'best' runs and is typical of that whole set—figures 12a and b pertain to 20 °C ambient and figures 12c and d to 10 °C. For each curve, the ordinate is $\log_e F(f)$ and the abscissa is (frequency)$^{\frac{1}{2}}$. The simplified response curve predicted by (1.1) is a straight line in this system and the particular straight lines obtained from the single sinusoid approximation for each of these runs are also shown. The short vertical line with the bars on the top and bottom represents a 10 % variation. It is found that the experimental points give a fit within 5 % of the straight line from about 75 to 625 Hz. Above this upper frequency the experimental points fall increasingly below the straight line and usually possess considerable scatter. However, the accuracy of the spectral estimates in this frequency region is suspect because of small power levels (in spite of the pre-emphasis) and possible slight fluctuations in the plume during the experiment. We are therefore uncertain about the significance of the discrepancy at high frequencies. The experimental points also tend to fall below the theoretical curve for frequencies less than 75 Hz. This is evidence of a real limitation in experimental technique because at zero frequency where the difference is greatest a response of unity is guaranteed by equations (A 1) to (A 3).

This droop may be due to our inability to sample the 'tail' of the signal from the platinum thermometer accurately enough and long enough. It may also be caused in part by a slowly varying ambient temperature during the experiment and by

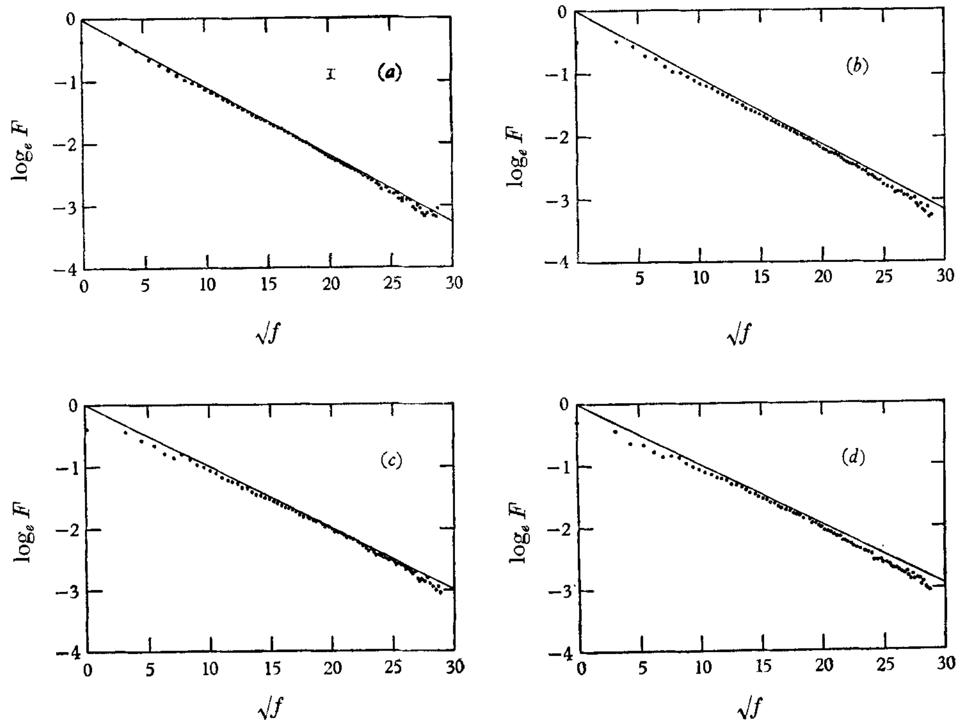

FIGURE 12. $\text{Log}_e F$ *vs.* (frequency)$^{\frac{1}{2}}$. The straight lines are graphs of $\log_e F = -\{(\Delta^2/D)\pi f\}^{\frac{1}{2}}$, where Δ is the diffusion-layer thickness determined separately for each graph, D is the thermal diffusivity and f is the frequency in Hertz. The points represented by the dots are the experimental determinations of F obtained from equations (A 1), (A 2) and (A 3). In (12*a*) the length of the short vertical line represents a variation of 10 % in F. (*a*) Velocity = 135·59 cm/sec; ambient temperature = 21·2 °C. (*b*) Velocity = 135·86 cm/sec; ambient temperature = 21·35 °C; (*c*) Velocity = 177·30 cm/sec; ambient temperature = 10·94 °C; (*d*) Velocity = 175·90 cm/sec; ambient temperature = 10·38 °C.

an ambient temperature difference across the plume. Whatever the reason, we are confident that this droop is an artifact of the experimental technique and not of the frequency response of the probe.

In spite of these discrepancies, the result (1.1) of the diffusion-layer model appears to be correct to within 5 % up to 625 Hz and the simple test method based on the single sinusoid approximation to the plume profile is a satisfactory procedure for routine use.

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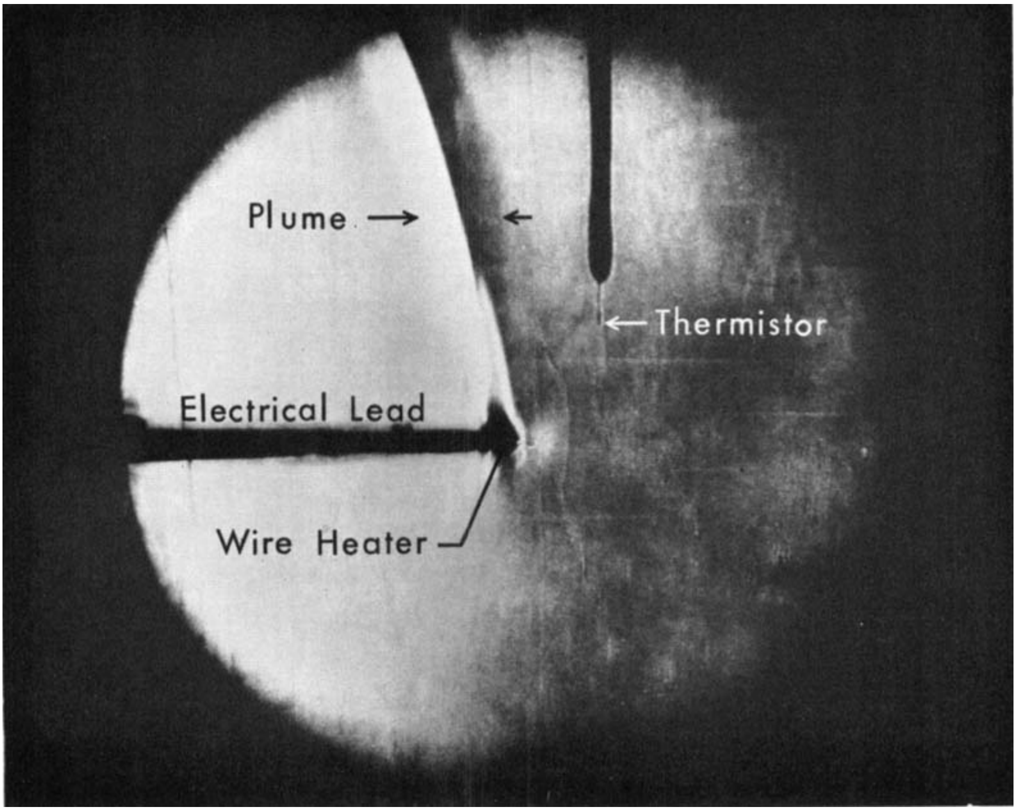

FIGURE 2. Schlieren image of the plume rising from a wire heater.

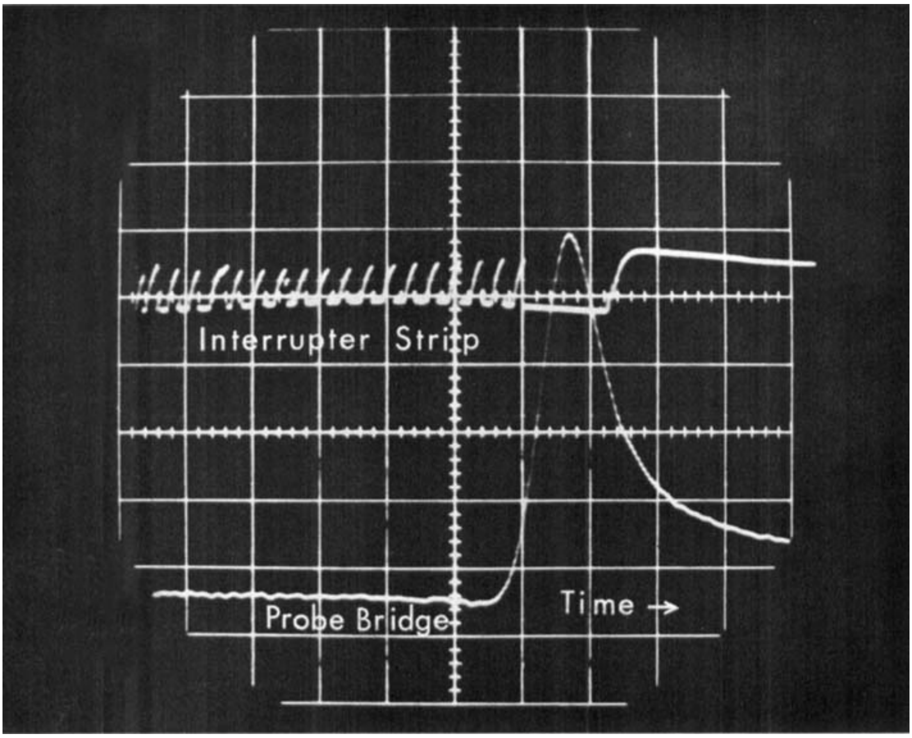

FIGURE 4. Oscilloscope traces of probe bridge output and interrupter strip output during a plume penetration.

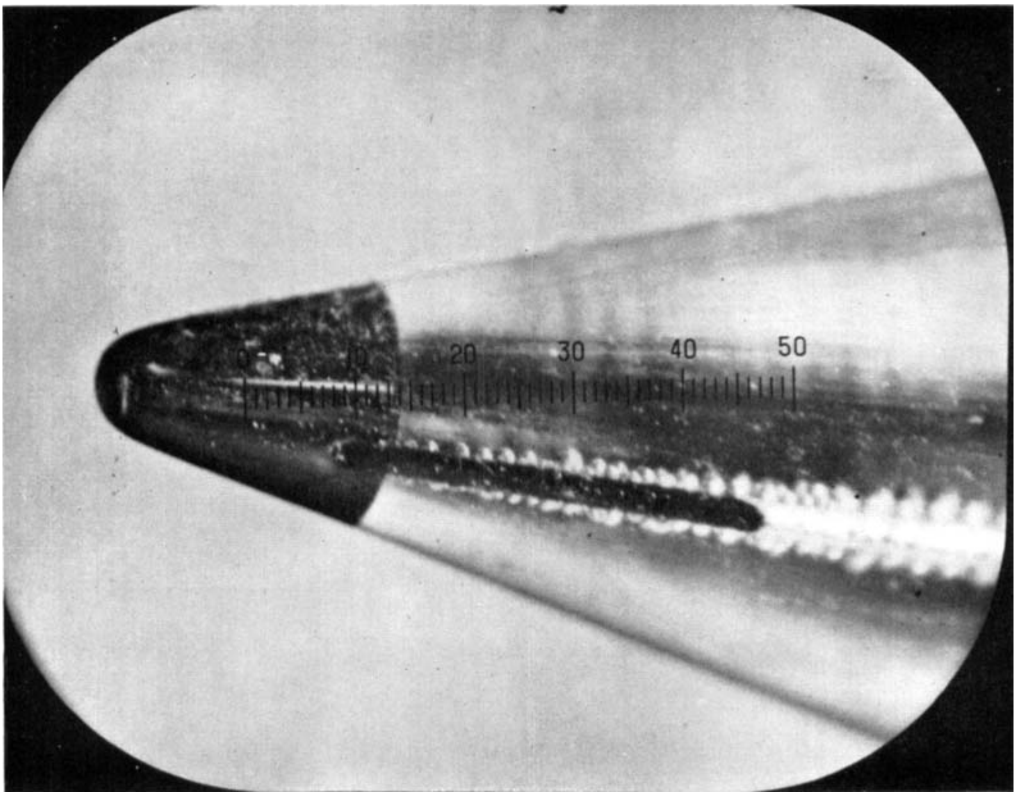

FIGURE 10. The probe used for the measurements reported in the appendix. The sensitive thin film is the dark area covering the tip of the glass cone. The long narrow dark tab is the edge of one of the connecting wires. The row of bubbles is encased in the glass around one of the connecting wires. Each division of the graticule is 0·033 mm.